WAVELENGHTS OF SPECTRAL LINES OF HYDROGEN ATOM AND FORGOTTEN BOHR FORMULA

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ABSTRACT

With use of original Bohr model, corrected in accordance with Archimedes and Ampere laws, was obtained forgotten Bohr formula, which was found to be consistent with experimental data.

ORIGINAL BOHR MODEL OF HYDROGEN ATOM

In accordance with Coulomb law, attraction force between proton and electron is

F_{colomb} , N = $\{c^2/10^7\}e^2/r_e^2$ (1)

Here c = 299 792 458 m/s is speed of light in vacuum, e = 1.602 176 620 8×10^{-19} Coulombs is elementary charge, r_e is radius of electron orbit. Centrifugal force is given by:

$F_{centrifugal}$, N = $m_e v_e^2/r_e$ (2)

From equality $F_{colomb} = F_{centrifugal}$, there is relation:

$m_e v_e^2 = \{c^2/10^7\}e^2/r_e$ (3) or

$2E_{kin} = E_{colomb}$ (4)

From this relation, orbital velocity of electron is defined by:

$v_e = \{c^2/10^7\}e^2/r_e m_e v_e$ (5)

Bohr assumed equality:

$n\hbar = r_e m_e v_e$ (6)

Here n is level number.

With this guess, orbital velocity of electron at n level is

$v_{en} = \{c^2/10^7\}e^2/n\hbar$ = [2 187 691.262 716 m/s]/n (7)

Fine structure constant is then

$\alpha = v_{e1}/c = \{c/10^7\}e^2/\hbar$ = 1/137.03599914… = 0.0072973525662… (8)

Here v_{e1} is velocity of electron at 1^{st} level.

Radius or circular trajectory of electron is:

$$r_{en} = \{c^2/10^7\}e^2/m_e v_e^2 = n\hbar/m_e v_e = [0.529\ 177\ 210\ 564...\ Å]n^2 \quad (9)$$

From Eq (4), binding energy at 1st level E1 = ($E_{coulomb} - E_{ekin}$) = E_{ekin}:

$$E1 = E_{coulomb} - E_{kin} = 0.5E_{coulomb} = E_{ekin} = m_e v_e^2/2 = 2.179\ 872\ 32539\times10^{-18}\ J \quad (10)$$

Binding energies at higher levels:

$$E_n = E1/n^2 \quad (11)$$

Lines of ionization from n level (in vacuum):

$$\lambda(n\text{-}\infty) = ch/E_n = [911.267\ 050\ 38385\ Å]n^2 \quad (12)$$

Corresponding frequencies are:

$$\nu(n\text{-}\infty) = c/\lambda(n\text{-}\infty) = E_n/h = [3.289\ 841\ 961{*}10^{15}\ Hz]/n^2 \quad (13)$$

Specific frequency of hydrogen atom (number of electron rotations per second = Hz) is:

$$\nu_{At} = \{v_e/2\pi r_e\} = [6.579\ 683\ 922{*}10^{15}\ Hz]/n^3 \quad (14)$$

Interestingly, that resonance between atom and light arises at $\nu_{light}(n\text{-}\infty) = 0.5n\nu_{Atn}$, i.e. at $\nu(1\text{-}\infty) = 0.5\nu_{At,n1}$, at $\nu(2\text{-}\infty) = \nu_{At,n2}$, at $\nu(3\text{-}\infty) = 1.5\nu_{At,n3}$, at $\nu(4\text{-}\infty) = 2\nu_{At,n4}$, etc.

Frequency of translation between levels is defined by

$$\nu(n_1\text{-}n_2) = \nu(n_1\text{-}\infty) - \nu(n_2\text{-}\infty) = [3.289\ 841\ 961{*}10^{15}\ Hz]\{1/n_1^2 - 1/n_2^2\} \quad (15)$$

Wavelengths of spectral lines (in vacuum) may be then calculated from:

$$\lambda(n_1\text{-}n_2) = ch/(E_{n1}-E_{n2}) = [911.267\ 050\ 38385\ Å]/\{1/n_1^2 - 1/n_2^2\} \quad (16)$$

Here $1/R_H$ = 911. 267 050 38385 Å is reversed Rydberg constant. As may be seen in Tab 2, within ~ 0.0535 %, original Bohr model is consistent with measurements.

Tab 1 Fundamental constants (Mohr et al 2016)

Speed of light, c	299 792 458 m/s
Plank constant, h	$6.626\ 070\ 04\times10^{-34}$ J×s
Reduced Plank constant, $\hbar = h/2\pi$	$1.054\ 571\ 8\times10^{-34}$ J×s
Fine structure constant, α	1/137.03599914
Elementary charge, e	$1.602\ 176\ 620\ 8\times10^{-19}$ C
Mass of electron, m_e	$0.000\ 910\ 938\ 356\times10^{-27}$ kg
Mass of proton, m_p	$1.672\ 621\ 898\times10^{-27}$ kg
Ratio m_p/m_e	1 836.152 674
Ratio m_e/m_p	0.0005446170214
Rydberg constant, R_i	10 973 731.568 539 m^{-1}
Rydberg constant of ^1H, $R_H = R_i/(1+m_e/m_p)$	10 967 758.34 m^{-1}

Tab 2 Wavelengths of Spectral lines of Hydrogen atom in vacuum and prediction from Bohr model, and that multiplied by $(1+m_e/m_p)$.

Trans-lation	Compilation of experimental data from Kramida (2010)[a]	Bohr model wavelengths, Å (Eq 16)	Error, observed minus calculated, %	Bohr model wavelengths, Å multiplied by $(1+m_e/m_p)$ (Eq 17)	Error, observed minus calculated, %
Lyman series					
1-2	1215.6701	1215.022 734	+0.0533	1215.684 456	–0.001181
1-3	1025.7283	1025.175 432	+0.0539	1025.733 760	–0.000532
1-4	972.5167	972.018 187	+0.0513	972.547 565	–0.003174
1-5	949.7416	949.236 511	+0.0532	949.753 481	–0.001251
1-6	937.8136	937.303 252	+0.0544	937.813 723	–0.000013
1-7	930.7512	930.251 781	+0.0537	930.758 412	–0.000775
1-8	926.2493	925.731 607	+0.0559	926.235 776	+0.001460
1-9	923.1479	922.657 889	+0.0531	923.160 384	–0.001352
1-10	920.9468	920.471 768	+0.0516	920.973 073	–0.002853
1-11	919.3424	918.860 943	+0.0524	919.361 370	–0.002063
1-12	918.1253	917.639 547	+0.0529	918.139 309	–0.001526
1-∞	[911.752 353][b]	911.267 05038	+0.05326	911.763 34193	–0.001205
Balmer series					
2-3	6564.6046	6561.122 763	+0.0515	6564.696 062	–0.001393
2-4	4862.7087	4860.090 935	+0.0539	4862.737 824	–0.000599
2-5	4341.6930	4339.366 907	+0.0536	4341.730 200	–0.000857
2-6	4102.8923	4100.701 727	+0.0534	4102.935 039	–0.001042
2-7	3971.1977	3969.074 264	+0.0535	3971.235 889	–0.000962
2-8	3890.1666	3888.072 748	+0.0539	3890.190 259	–0.000608
2-9	3836.4844	3834.422 394	+0.0538	3836.510 686	–0.000685
2-10	3798.9880	3796.946 044	+0.0538	3799.013 925	–0.000682
2-11	3771.7047	3769.685 918	+0.0536	3771.738 953	–0.000908
2-12	3751.2163	3749.213 007	+0.0534	3751.254 893	–0.001028
2-13	3735.4314	3733.433 491	+0.0535	3735.466 783	–0.000947
2-14	3723.0040	3721.007122	+0.0537	3723.033 646	–0.000796
2-15	3713.0344	3711.042 287	+0.0537	3713.063 383	–0.000781
2-16	3704.9126	3702.926 427	+0.0536	3704.943 104	–0.000823
2-17	3698.2098	3696.227 053	+0.0536	3698.240082	–0.000819
2-18	3692.6013	3690.631 554	+0.0534	3692.641 535	–0.001090
2-19	3687.8800	3685.909 302	+0.0535	3687.916 711	–0.000995
2-20	3683.8708	3681.887 072	+0.0539	3683.892 291	–0.000583
2-21	3680.4162	3678.432 670	+0.0539	3680.436 007	–0.000538
2-22	3677.4224	3675.443770	+0.0538	3677.445 479	–0.000628
2-∞	[3647.022 802][b]	3645.068 20154	+0.05362	3647.053 36772	–0.000838

a: data from Tab. 10 and 11 in Kramida (2010), initially presented in the inversed centimeters, mostly with 7 digits.
b: averaged values, calculated from experimental data (this study).

As may be seen, deviation of original Bohr model from data is close to electron-to-proton mass ratio. Thus, multiplying wavelengths from original Bohr model by "adjusting factor" $(1+m_e/m_p)$, one may obtain much closer result (see last columns in Tab. 2):

$$\lambda(n1\text{-}n2) = [911.267\ 050\ 38385\ \text{Å}](1+m_e/m_p)/\{1/n_1^2 - 1/n_2^2\} =$$
$$= [911.763\ 341\ 93052\ \text{Å}]\ /\{1/n_1^2 - 1/n_2^2\} = \{1/R_H\}/\{1/n_1^2 - 1/n_2^2\} \quad (17)$$

Here $1/R_H$ = 911.763 341 93052 Å is reversed Rydberg constant for ^{1}H. Let us consider the origin of "adjusting factor" $(1+m_e/m_p)$.

BOHR MODEL WITH ROTATING PROTON

At the movement of electron and proton around the barycenter, the Archimedes law of lever should be satisfied:

$$|v_p/v_e| = r_p/r_e = m_e/m_p \quad (18)$$

From Eq (18), radius of proton orbit and its orbital velocity may be found from:

$$r_p = r_e m_e/m_p \quad (19)$$

$$|v_p| = |v_e| m_e/m_p \quad (20)$$

Note here that velocities of proton and electron are always opposite (see Fig 1).

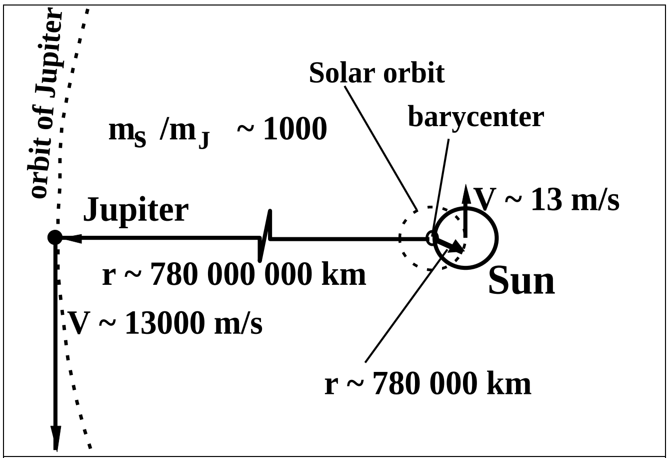

Fig. 1 Idealized orbit of Sun (neglecting tags from all planets except Jupiter).

Balance between Coulomb and centrifugal tags should be rewritten as:

$$m_e v_e^2/r_e = m_p v_p^2/r_p = \{c^2/10^7\}e^2/(r_e+r_p)^2 \quad (21)$$

This relation may be transformed to:

$$(1+m_e/m_p)*m_e v_e^2/(r_e + r_p) = (1+m_p/m_e)*m_p v_p^2/(r_e + r_p) = \{c^2/10^7\}e^2/(r_e+r_p)^2 \quad (22) \text{ or}$$

$$(1+m_e/m_p)*m_e v_e^2 = (1+m_p/m_e)*m_p v_p^2 = \{c^2/10^7\}e^2/(r_e+r_p) \quad (23)$$

Thus, orbital velocities of electron and proton are

$$v_e = \{c^2/10^7\}e^2/\{(r_e+r_p)m_e v_e\}/(1+m_e/m_p) \quad (24)$$

$$v_p = \{c^2/10^7\}e^2/\{(r_e+r_p)m_p v_p\}/(1+m_p/m_e) = v_e m_e/m_p \quad (25)$$

Since equality $n\hbar = m_e v_e r_e \approx 1836\times m_p v_p r_p$ seems to be doubtful, let as assume the following relation:

$$n\hbar = (r_e + r_p)\times m_e\times v_e = (r_e + r_p)\times m_p\times v_p \quad (26)$$

Note here that $(r_e + r_p)$ is distance between electron and proton, like in Eq (6).

Thus, orbital velocities of electron and proton may be calculated from:

$v_e = \{c^2/10^7\}e^2/\{n\hbar(1+m_e/m_p)\} = [2186500.457\ 349\ \text{m/s}\ ]/n$ (27)

$v_p = \{c^2/10^7\}e^2/\{n\hbar(1+m_p/m_e)\} = v_e m_e/m_p = [1190.805\ 366\ \text{m/s}]/n$ (28)

Orbital radii may be then calculated from

$r_e + r_p = n\hbar/m_e v_e = n\hbar/m_p v_p = [0.529\ 465\ 40948\ \text{Å}]n^2$ (29)

$r_e = (r_e+r_p)/(1+m_e/m_p) = [0.529\ 177\ 21056\ \text{Å}]n^2$ (30)

$r_p = (r_e+r_p)/(1+m_p/m_e) = r_e m_e/m_p = [0.000288\ 19892\ \text{Å}]n^2$ (31)

Kinetic energies of electron and proton are:

$E_{e\text{-}kin} = m_e v_e^2/2 = [2.177\ 499\ 8725{*}10^{-18}\ \text{J}]/n^2$ (32)

$E_{p\text{-}kin} = m_p v_p^2/2 = \{m_e/m_p\}E_{ekin} = [0.001\ 185\ 9035{*}10^{-18}\ \text{J}]/n^2$ (33)

Thus, total kinetic energy is:

$E_{kin} = E_{ekin} + E_{pkin} = E_{ekin}(1+m_e/m_p) = E_{pkin}(1+m_p/m_e) = [2.178\ 685\ 776{*}10^{-18}\ \text{J}]/n^2$ (34)

Coulomb energy is

$$E_{Coulomb} = \{c^2/10^7\}e^2/(r_e+r_p) = \{c^2/10^7\}e^2 m_e v_e/n\hbar = \{c^2/10^7\}e^2 m_p v_p/n\hbar = = [4.357\ 371\ 552{*}10^{-18}\ \text{J}]/n^2 \quad (35)$$

Thus, binding energy at 1st level is:

$E1 = E_{Coulomb} – E_{ekin} – E_{pkin} = 0.5E_{Coulomb} = E_{kin} = 2.178\ 685\ 776{*}10^{-18}\ \text{J}$ (36)

Thus, wavelengths may be calculated

$\lambda(1\text{-}\infty) = ch/E1 = 1/R_H = 911.763\ 341\ 93052\ \text{Å}$ (37)

$\lambda(n\text{-}\infty) = (ch/E1)n^2 = [911.763\ 341\ 93052\ \text{Å}]n^2$ (38)

$\lambda(n_1\text{-}n_2) = [911.763\ 341\ 93052\ \text{Å}]/(1/n_1^2 – 1/n_2^2)$ (39)

As may be seen, wavelengths from Bohr model with rotating proton (Eq 39) are equal to original Bohr model wavelengths, multiplied by the factor $(1+m_e/m_p)$, Eq (17).

However there is systematic overestimation by ~0.001 % (see Tab 2). Thus, multiplying Eq (39) by factor 1/1.00001, one may obtain almost exact fit to data. So, let us consider the origin of “adjusting factor” ~ 1/1.00001.

AMPERE LAW AND THEORY OF RELATIVITY

In accordance with Ampere law, magnetic force between element of electric current and parallel infinite current is defined by (see Fig 2):

$F/\Delta L_1$, N/m $= (1/10^7){*}2I_1I_2/r$ (at $I_1 \parallel I_2 \perp r$) (40)

Wire with electric current I_1 = 1 A and its length ΔL = 1 m interacts with parallel infinite electric current I_2 = 1 A, located at dictance r = 1 m, with force $F = 2*10^{-7}$ N. This is attraction if currents flow in the same direction, and it is repulsion if currents flow in opposite directions.

From this relation, one may guess differential Ampere law for parallel elements of current, located on perpendicular to both currents:

$$F, N = (1/10^7)*(I_1dL_1)*(I_2dL_2)/r^2 \qquad \text{(valid at } I_1 \parallel I_2 \perp r\text{, and } dL_1, dL_2 << r) \qquad (41)$$

Original Amper law (Eq. 40), may be obtained via integration of tags from elements of currents in parallel and infinite wire with electric current I_2.

$$F, N = (1/10^7)*(I_1dL_1)\, 2\int_{L=0}^{L=\infty}(I_2dL)\sin(\alpha)/(r^2 + L^2) = (1/10^7)*(I_1dL_1)\, 2\int_{L=0}^{L=\infty}(I_2dL)r/(r^2 + L^2)^{1.5} =$$

$$= (1/10^7)*2(I_1dL_1)*I_2/r \qquad (42)$$

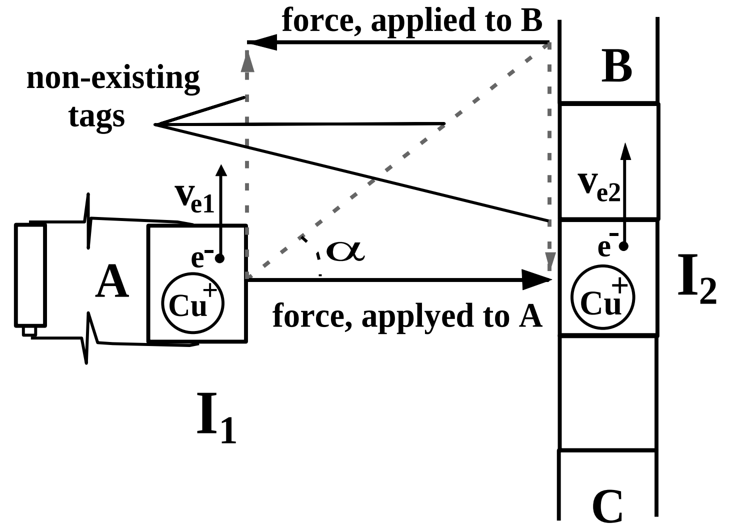

Fig. 2 Scheme of interaction between element of current I_1 (A) and element of infinite current I_2 (B).

Factor 2 arises here dew to integration from L = 0 to + ∞, and from L = 0 to – ∞, $\sin(\alpha) = r/(r^2+L^2)^{0.5}$ is sinus of angle between distance and perpendicular to wires (see Fig 2). Resulting tags are just projections of “hypothetic total tag” onto perpendicular to wires. So, Eq. (38 = 39) is sum of tags, orthogonal to wires with current. In classic approach, magnet tag is always orthogonal to velocity of electrons, whereas parallel tags simply do not exist (see Fig. 2). Even if they exist, parallel tags from “upper” element B and “lower” element C of infinite current I_2 eliminate each over. So, let us follow the classic approach.

Element of current IdL may be represented by:

$$IdL, \text{Coulomb*m/s} = [e*D_e, \text{Coulombs/mm}^3][v_e, \text{mm/s}][S, \text{mm}^2][dL, \text{m}]$$
$$= n_e*e*[v_e, \text{m/s}] \qquad (43)$$

Here e is elementary charge (1.602 176 620 8×10^{-19} Coulomb), D_e is density of electrons, v_e is velocity of electrons, S and dL are cross-section area and length of wire, n_e is number of electrons in element of current.

For instance, density of copper is 8.932 g/cm^3 = 8.932 mg/mm^3. From molar mass of cupper, 63.546 g/mol and Avogadro number, density of electrons in cupper is D_e = (8.932 mg/mm^3/63546 mg/mol)*6.022045*10^{23} mol^{-1} = 8.464562*10^{19} electrons/mm^3. Thus, charge density of electrons in cupper (as well as ions Cu^+) is $e*D_e$ = 13.5617 Coulomb/mm^3.

From these estimates, velocity of electrons in cupper wires is:

$$v_e\text{, mm/s} = [I\text{, A}]/\{[13.5617 \text{ Coulombs/mm}^3][S\text{, mm}^2]\} \quad (44)$$

For instance, at cross-section of wire S = 1 mm^2, and electric current I = 1 A, velocity of electrons is only 0.073737 mm/s.

So on, differential Ampere law (Eq 38) may be rewritten as

$$F\text{, N} = (1/10^7)*(n_1*n_2)e^2*[v_{e1}\text{, m/s}][v_{e2}\text{, m/s}]/[r\text{, m}]^2 \qquad (\text{at } v_{e1} \parallel v_{e2} \perp r) \quad (45)$$

Here n_1 and n_2 are numbers of electrons (= numbers of ions Cu^+) in elements of electric currents, v_{e1} and v_{e2} are velocities of electrons in wires.

However there is a problem: if observer walks (together with system of coordinates) along wires with velocity v_{e1} or v_{e2}, interaction force between wires should be observed at zero! This mystery is origin of numerous hypotheses, including Theory of Relativity.

However there is a simple way to solve this puzzle (Pivovarov, 2014). Indeed, Eq (45) may be rewritten as (also at $v_{e1} \parallel v_{e2} \perp r$):

$$F(n_1e^-{}_1 \text{ and } n_2e^-{}_2) = -(c^2/10^7)\{n_1n_2e^2/r^2\}[1 + 0.5\{(v_{e1}-v_{e2})^2/c^2\}] =$$
$$= -(c^2/10^7)\{n_1n_2e^2/r^2\}[1 + 0.5\{(v_{e1}{}^2 - 2v_{e1}v_{e2}+v_{e2}{}^2)/c^2\}] \quad (46)$$

$$F(n_1e^-{}_1 \text{ and } n_2Cu^+{}_2) = +(c^2/10^7)\{n_1n_2e^2/r^2\}[1 + 0.5\{(v_{e1})^2/c^2\}] \quad (47)$$

$$F(n_1Cu^+{}_1 \text{ and } n_2e^-{}_2) = +(c^2/10^7)\{n_1n_2e^2/r^2\}[1 + 0.5\{(v_{e2})^2/c^2\}] \quad (48)$$

$$F(n_1Cu^+{}_1 \text{ and } n_2Cu^+{}_2) = -(c^2/10^7)n_1n_2e^2/r^2 \quad (49)$$

Thus, total tag between elements of currents is:

$$F_{SUM} = (1/10^7)\{n_1n_2e^2/r^2\}(v_{e1}*v_{e2}) \qquad (\text{at } v_{e1} \parallel v_{e2} \perp r) \quad (50)$$

Here, $(v_{e1} - v_{e2})$ is velocity of electron in 1st wire relatively to electron in 2nd wire, v_{e1} is velocity of electron in 1st wire relatively to ion Cu^+ in 2nd wire, and v_2 is velocity of electron in 2nd wire relatively to ion Cu^+ in 1st wire. As may be seen, this theory of relativity needs no corrections on distortion of time and space.

So on, electromagnetic force between proton and electron at circular trajectory in atom of hydrogen is:

$$F_{CM} = (c^2/10^7)\{e^2/r^2\}[1 + 0.5\{v^2/c^2\}] \qquad \text{or, may be} \quad (51)$$

$$F_{CM} = (c^2/10^7)\{e^2/r^2\}[1 + \{v^2/c^2\}]^{0.5} \qquad \text{or, may be} \quad (52)$$

$$F_{CM} = (c^2/10^7)\{e^2/r^2\}/[1 - \{v^2/c^2\}]^{0.5} \qquad \text{or etc} \quad (53)$$

Here v is velocity of electron relatively to proton or, the same, velocity of proton relatively to electron, and r is distance between electron and proton. Note here, that at circular orbit of electron, condition $v \perp r$ is always satisfied. At 1st level of hydrogen atom, velocity of electron is about 2188 km/s, and relation $0.5(v/c)^2 \approx 0.5\alpha^2 \approx 0.00002663$ is rather small. Because of this, all variants (Eqs 51, 52, 53) are almost identical in application to hydrogen atom. So, let as try most simple first variant (Eq. 51).

BOHR MODEL WITH ROTATING PROTON AND MAGNET TAG AND FOGOTTEN BOHR FORMULA

At circular orbits of electron and proton, there is equality between centrifugal and electromagnetic forces:

$$m_e v_e^2/r_e = m_p v_p^2/r_p = (c^2/10^7)\{e^2/(r_e+r_p)^2\}[1 + 0.5\{(v_e+v_p)^2/c^2\}] \quad (54)$$

Appling $r_e = (r_e+r_p)/(1+m_e/m_p)$ and $r_p = (r_e+r_p)/(1+m_p/m_e)$, this relation may be reduced to:

$$(1+m_e/m_p)m_e v_e^2 = (1+m_p/m_e)m_p v_p^2 = (c^2/10^7)\{e^2/(r_e+r_p)\}[1 + 0.5\{(v_e+v_p)^2/c^2\}] \quad (55)$$

Here r_e+r_p is distance between electron and proton, v_e+v_p is velocity of electron relatively to proton (or, the same, velocity of proton relatively to electron). So, orbital velocities of electron and proton at 1st level are:

$$v_{e1} = [(c^2/10^7)e^2/\hbar/(1+m_e/m_p)]\,[1 + 0.5\{(v_e+v_p)^2/c^2\}] =$$

$$= [2186500.457\ 349\ \mathrm{m/s}]*[1 + 0.5\{(v_e+v_p)^2/c^2\}] \quad (56)$$

$$v_{p1} = [(c^2/10^7)e^2/\hbar/(1+m_p/m_e)]\,[1 + 0.5\{(v_e+v_p)^2/c^2\}] = v_{e1}m_e/m_p \quad (57)$$

Solving these relations iteratively, one may obtain velocities of electron and proton at 1st level:

$$v_{e1} = 2186558.677\ 505\ldots\ \mathrm{m/s} \quad (58)$$

$$v_{p1} = 1190.837\ 074\ldots\ \mathrm{m/s} \quad (59)$$

Velocity of electron relatively to proton (or, velocity of proton with respect to electron) at 1st level is (see Fig 1):

$$v_{e1}+v_{p1} = v_{e1}(1+m_e/m_p) = 2187\ 749.514\ 579\ldots\ \mathrm{m/s} \quad (60)$$

Distance between electron and proton at 1st level is

$$(r_{e1}+r_{p1}) = \hbar/m_e v_{e1} = \hbar/m_p v_{p1} = 0.529\ 451\ 311\ 729\ 572\ldots\ \text{Å} \quad (61)$$

Radii of electron and proton orbits at 1st level:

$$r_{e1} = (r_{e1}+r_{p1})/(1+m_e/m_p) = 0.529\ 163\ 120\ 487\ 063\ldots\ \text{Å} \quad (62)$$

$$r_{p1} = (r_{e1}+ r_{p1})/(1+m_p/m_e) = r_{e1}m_e/m_p = 0.000\ 288\ 191\ 242\ 509\ldots\ \text{Å} \quad (63)$$

Similarly, velocities of electron and proton at level n may be calculated from:

$$v_{en} = (1/n)[2186500.457\ 349\ \mathrm{m/s}]*[1 + 0.5\{(v_{en}+v_{pn})^2/c^2\}] \quad (64)$$

$$v_{pn} = v_{en}m_e/m_p \quad (65)$$

Kinetic energies of electron and proton at 1st level may be calculated from:

E_{ekin} (n=1) = $m_e v_e^2/2$ = 2.177 615 762 768 852 $*10^{-18}$ J (66)

E_{pkin} (n=1) = $m_p v_p^2/2 = E_{ekin} m_e/m_p$ = 0.001 859 666 104 505 $*10^{-18}$ J (67)

Total kinetic energy at 1st level is:

E_{kin} (n=1) = $E_{ekin} + E_{pkin} = E_{ekin}(1+m_e/m_p)$ = 2.178 801 8017… $*10^{-18}$ J (68)

Kinetic energy at others levels may be calculated from:

$E_{kin}(n) = 0.5 m_e v_{en}^2 [1+m_e/m_p]$ (69)

Coulombic Energy at 1st level is:

$E_{Coulomb}$ (n=1) = $(c^2/10^7)\{e^2/(r_e+r_p)\} = (c^2/10^7)\{m_e v_{e1} e^2/\hbar\}$ = 4.357 487 5762… $*10^{-18}$ J (70)

$E_{Coulomb}(n) = (c^2/10^7)\{m_e v_{en} e^2/n\hbar\}$ (71)

Magnet tag is defined by:

F_{Magnet} (n=1) = $(0.5/10^7)e^2(v_{e1}+v_{p1})^2/(r_{e1}+r_{p1})^2 = (0.5/10^7)\{e^2(v_{e1}+v_{p1})^2(v_{e1}m_e/\hbar)^2\}$ (72)

F_{Magnet} (n=2) ≈ $(0.5/10^7)e^2(0.5v_{e1}+0.5v_{p1})^2/(4r_{e1}+4r_{p1})^2 = (0.5/10^7)\{e^2(v_{e1}+v_{p1})^2/(r_{e1}+r_{p1})^2\}/2^6$ (73)

F_{Magnet} (n=3) ≈ $(0.5/10^7)e^2(v_{e1}/3+v_{p1}/3)^2/(9r_{e1}+9r_{p1})^2 = (0.5/10^7)\{e^2(v_{e1}+v_{p1})^2/(r_{e1}+r_{p1})^2\}/3^6$ (74)

As may be seen, Magnet tag drops as $1/n^6$, or as $1/(r_e+r_p)^3$. Integer of $1/x^3$ is $-0.5/x^2$. Thus, hypothetical "Magnet energy" drops as $1/n^4$ or $1/r^2$ and may be guessed from:

$E_{Mn} = (0.25/10^7)\{e^2(v_{en}+v_{pn})^2\}/(r_{en}+r_{pn}) = (0.25/10^7)\{e^2(v_{en}+v_{pn})^2\}v_{en}m_e/(n\hbar) \sim E_{M1}/n^4$ (75)

"Magnet energy" at 1st level is:

$E_{M1} = (0.25/10^7)\{e^2(v_{e1}+v_{p1})^2\}/(r_{e1}+r_{p1}) = (0.25/10^7)\{e^2(v_{e1}+v_{p1})^2\}v_{e1}m_e/\hbar$ =
= 0.000 058 0136…$*10^{-18}$ J (76)

Thus binding energy at 1st level is:

$E1 = E_{C1} + E_{M1} - E_{kin1}$ = 2.178 743 7881…$*10^{-18}$ J (77)

$\lambda(1\text{-}\infty) = hc/E1$ = 911.739 064 967 6533… Å (78)

Close approximation to present model (exact within 8 digits) is "formula, suggested by Dr. Bohr in letter to Prof. Fowler" (citation from Curtis, 1914, with parameter "k" instead $0.5\alpha^2$):

$\lambda(n_1\text{-}n_2) = 1/R_H/(1/n_1^2-1/n_2^2)/\{1+0.5\alpha^2(1/n_1^2+1/n_2^2)\}$ =
= [911.763 341 93052…Å]$/\{1/n_1^2-1/n_2^2+0.5\alpha^2(1/n_1^4-1/n_2^4)\}$ (79)

$\lambda(n\text{-}\infty)$ = [911.763 341 93052…Å]$/(1/n^2+0.5\alpha^2/n^4)$ (80)

Here 911.763 341 93052…Å = $1/R_H$ is reversed Rydberg constant for hydrogen, $\alpha = \{c/10^7\}e^2/\hbar = 1/137.03599914…$ is fine structure constant.

Original Bohr letter (15 April 1914) with parameter π^2e^4/h^2c^2 (CGSE units: e ~ 4.8*10^{-10} electric units; h ~ 6.6*10^{-27} erg*s; c ~ 3*10^{10} cm/s) = $\pi^2c^2e^4/10^{14}h^2$ (in SI units, see Tab 1) = $0.25\alpha^2$ instead $0.5\alpha^2$ in Eq 79, may be found in Hoyer (1981). In response to Bohr (20 April 1914), Prof Fowler reported experimental estimations for this parameter, 0.00003 to 0.00004, which is closer to $0.5\alpha^2$ = 0.000026625677…, as guessed in present study from Ampere law.

It should be noted that Eq (79) was deduced by Bohr from mass defect. However, this was just a commentary. So, let us deduce Eq (79) from mass defect. Taking binding energy from Eq (36), $E_n = chR_H/n^2$ = [2.178685776*10^{-18} J]/n^2, mass defect is $\Delta m_n = E_n/c^2$ = [2.424114851*10^{-35} kg]/n^2. Thus, assuming that this is mass defect of electron, kinetic energy should be slightly reduced, and thus, binding energy should slightly enlarged by $\Delta m_n v_{en}^2/2 = 0.5E_n v_{en}^2/c^2 = 0.5E_{n1}(v_{en1}/c)^2/n^4 = 0.5E_{n1}\alpha^2/n^4$. Thus, dissociation lines may be calculated as $\lambda(n\text{-}\infty) = ch/(E_{n1}/n^2 + 0.5E_{n1}\alpha^2/n^4)$ = Eq (80). Transition lines may be calculated as $\lambda(n_1\text{-}n_2) = 1/(1/\lambda_{n1\text{-}\infty} - 1/\lambda_{n2\text{-}\infty})$ = Eq. (79). As may be seen, both models, originating from mystical Ampere law give the same result. So, may be, mass defect arises dew to magnet force.

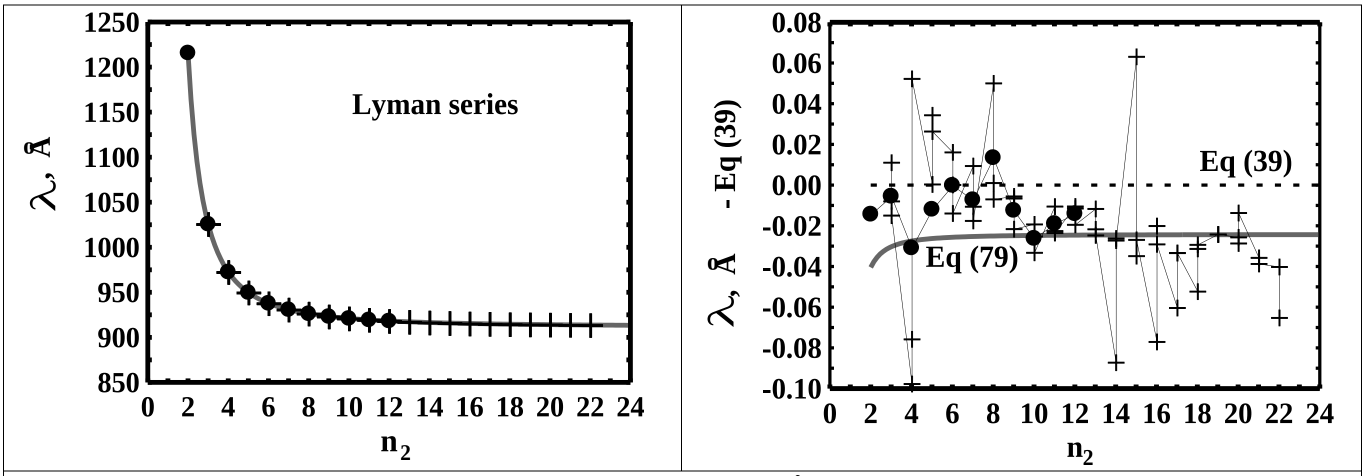

Fig. 3 Vacuum wavelengths of hydrogen Lyman series (Å) and deviation (right panel) from Bohr model with rotating proton Eq (39). Black points: "critical compilation of experimental data" (Kramida 2010); crosses: lines from 3 independent Solar spectra from Parenti et al (2005). Solid curve is Bohr model with rotating proton and magnet tag (Eq. 79).

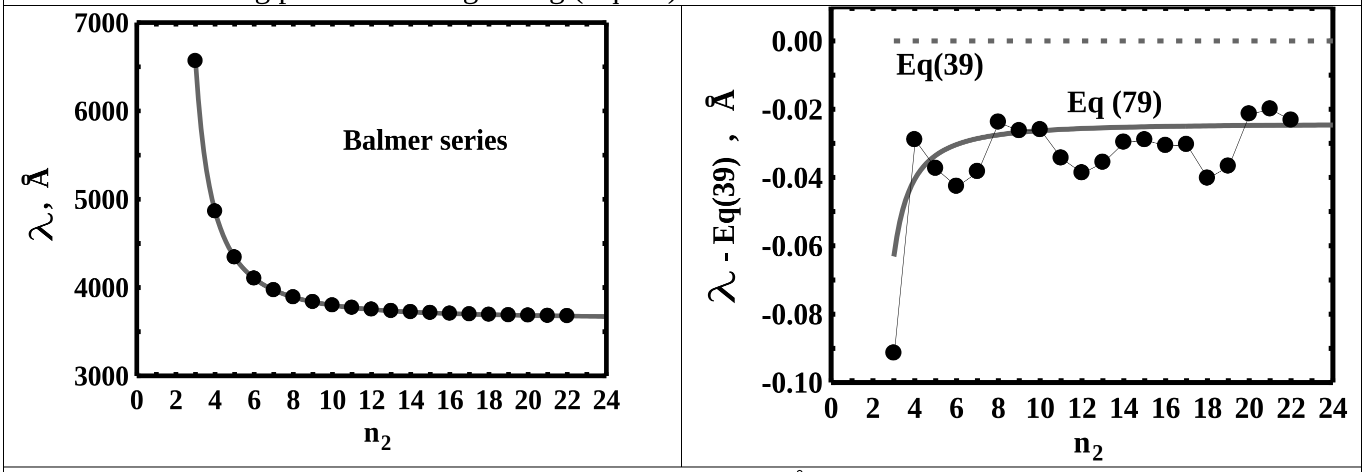

Fig. 4 Vacuum wavelengths of hydrogen Balmer series (Å) and deviation (right panel) from Bohr model with rotating proton Eq (39). Black points: "critical compilation of experimental data" (Kramida 2010). Solid curve is Bohr model with rotating proton and magnet tag (Eq. 79).

However, additional binding energy causes additional mass defect, which also contributes into binding energy and closer approximation to “magnet model” (exact within 12 digits) is:

$$\lambda(n_1\text{-}n_2) = [911.763\ 341\ 93052\ \text{Å}]/\{1/n_1^2-1/n_2^2 +0.5\alpha^2(1/n_1^4-1/n_2^4) +0.5\alpha^4(1/n_1^6-1/n_2^6)\} \quad (81)$$

$$\lambda(n\text{-}\infty) = [911.763\ 341\ 93052\ \text{Å}]/\{1/n^2 + 0.5\alpha^2/n^4 + 0.5\alpha^4/n^6\} \quad (82)$$

Calculated wavelengths given in Tabs. 3 and 4. It should be noted, that difference between Eqs (79) and exact solution is almost negligible: 0.000 002 262 Å at transition 1-2 and 0.000 001 293 Å at transition 1-∞ (Lyman series); 0.000 000 9552 Å at transition 2-3, and 0.000 000 3232 Å at transition 2-∞ (Balmer series), etc. So, there seems no significant sense in Eq. (81). Nevertheless, Eq.(81) was applied here to reproduce 6^{th} sign after comma in Tabs 3-5.

Fig. 3 (right) shows the difference between Lyman wavelengths of Bohr model with rotation of proton (Eq. 39) and experimental data (Parenti et al, 2005; Kramida, 2010). As may be seen, large and wide Lyman lines (up to n_2 = 9) seems to be equally consistent with both model (Eq. 39 or 79). However, small and fine lines (n_2 = 10 ÷ 22) seems to be consistent with Eq (79). As may be seen in Fig. 4 (right), Eq. (79) is well consistent with Balmer series data.

Tab. 3 Lyman series, vacuum wavelengths (Å).

Transition	Compilation of experimental data from Kramida (2010) (a)	Lyman series in Solar spectrum measured with automatic spacecraft (emission lines), Parenti et al, 2010 (3 spectra: 1; 2; 3)	Bohr model with rotation of proton and magnet tag (Eq. 81)
1-2	1215.670 (±0.002)	(b) ; (b) ; (b)	1215.643 994
1-3	1025.728 (±0.003)	1025.745; 1025.719; 1025.726	1025.703 414
1-4	972.517 (±0.015)	972.450; 972.472; 972.600	972.520 051
1-5	949.742 (±0.004)	949.754; 949.788; 949.780	949.727 181
1-6	937.814 (±0.014)	937.830; 937.814; 937.800	937.788 059
1-7	930.751 (±0.014)	930.768; 930.748; 930.741	930.733 123
1-8	926.249 (±0.014)	926.286; 926.237; 926.226	926.210 728
1-9	923.148 (±0.014)	923.155; 923.154; 923.139	923.135 500
1-10	920.947 (±0.014)	920.954; 920.948; 920.940	920.948 305
1-11	919.342 (±0.014)	919.351; 919.339; 919.338	919.336 688
1-12	918.125 (±0.014)	918.129; 918.128; 918.120	918.114 693
1-13	-	917.179; 917.169; 917.166	917.165 939
1-14	-	916.352; 916.413; 916.412	916.414 525
1-15	-	915.897; 915.799; 915.807	915.809 220
1-16	-	915.262; 915.319; 915.310	915.314 417
1-17	-	914.869; 914.896; 914.896	914.904 741
1-18	-	914.534; 914.555; 914.557	914.561 711
1-19	-	914.272; 914.272; 914.272	914.271 606
1-20	-	914.023; 914.020; 914.035	914.024 064
1-21	-	(c) ; 913.800; 913.797	913.811 144
1-22	-	(c) ; 913.611; 913.586	913.626 673

(a): data from Tab. 11 in Kramida (2010), initially, in reversed cm, with 6-7 digits.
(b): Line (~1215.6±0.5 Å) was ~ 2 times stronger than upper limit of detector’s scale.
(c): Lines were not distinguished dew to overlap with nearest H lines.

Tab 4. Balmer series, vacuum wavelengths (Å).

Transition	Compilation of experimental data from Kramida (2010) (a)	Bohr model with rotation of proton and magnet tag (Eq. 81)
2-3	6564.6046 (±0.03)	6564.632 943
2-4	4862.7087 (±0.04)	4862.697 363
2-5	4341.6930 (±0.006)	4341.696 675
2-6	4102.8923 (±0.007)	4102.904 693
2-7	3971.1977 (±0.006)	3971.207 297
2-8	3890.1666 (±0.006)	3890.162 746
2-9	3836.4844 (±0.006)	3836.483 887
2-10	3798.9880 (±0.006)	3798.987 625
2-11	3771.7047 (±0.006)	3771.713 017
2-12	3751.2163 (±0.006)	3751.229 229
2-13	3735.4314 (±0.006)	3735.441 329
2-14	3723.0040 (±0.006)	3723.008 358
2-15	3713.0344 (±0.006)	3713.038 228
2-16	3704.9126 (±0.006)	3704.918 057
2-17	3698.2098 (±0.006)	3698.215 124
2-18	3692.6013 (±0.006)	3692.616 651
2-19	3687.8800 (±0.006)	3687.891 890
2-20	3683.8708 (±0.006)	3683.867 524
2-21	3680.4162 (±0.006)	3680.411 286
2-22	3677.4224 (±0.006)	3677.420 798

(a): data from Table 10 in Kramida (2010), initially in reversed cm, with 7 digits

AIR SPECTRA

It should be also noted that observer fill bad in vacuum, and most of data were measured at atmospheric conditions (except Lyman series, because of aggressive behavior of oxygen in this spectral diapason). Speed of light in atmospheric air is smaller by ~ 83 km/s. In result, wavelengths also become shorter:

$$\lambda_{Air} = c_{Air}/\nu = \lambda_{vac}/n_{Air} \qquad (83)$$

Here $n_{Air} = c/c_{Air}$ is refractive index of air (~ 1.000277). In accordance with Ciddor (1996), refractive index of dry air with 450 ppm CO_2 may be calculated from:

$$n_{Air} = 1+f\{0.05792105/(238.0185–1/[\lambda_{vac}, \mu m]^2)+0.00167917/(57.362–1/[\lambda_{vac}, \mu m]^2)\} \qquad (84)$$

Here $f = q[P, Atm]*288.15/(t^oC+273.15)$, q is correction on humidity and CO_2 content (here, q = 1: see Ciddor, 1996, for details). Note here that t^oC, and P, Atm are temperature and pressure of air between prism (or grating) and detector, or inside of monochromator.

In Tab. 5, experimental wavelengths of hydrogen Balmer series, measured in air (Curtis, 1914, Wood, 1922, Perepelitsa and Pepper, 2006) are compared with Bohr model with rotation of proton and magnet tag, adjusted to air pressure 1 Atm and various temperatures 0, 15, 30^{o}C.

As may be seen in Fig 6, wavelengths of Balmer series lines in air are 1-2 Å smaller then in vacuum, whereas observers prefer ambient temperature +15°C and 1 Atm, or, may be, + 20°C at pressure 1.01735 Atm.

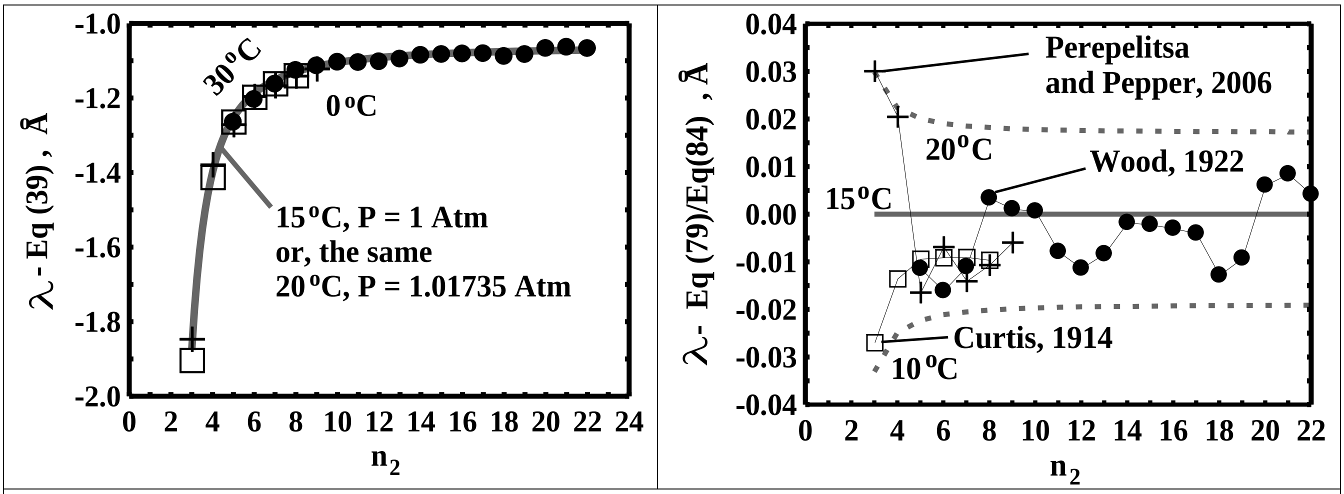

Fig. 5 Difference between Balmer wavelengths, measured in air and vacuum (Eq. 79; left) and (right) difference between data measured in air and Eq(79)/Eq(84) at 15°C and 1 Atm. Solid curve: 15°C, 1 Atm (or, 20°C and 1.01735 Atm). Dashed curves: various temperatures and 1 Atm. Points: data from Wood, 1922, cubes: Curtis, 1914, crosses: Perepelitsa and Pepper, 2006.

Tab. 5 Balmer lines of hydrogen, measured in air (Å).

Transi-tion	Perepelitsa and Pepper, 2006	Wood, 1922	Curtis, 1914	Bohr model with rotation of proton, and magnet tag, at 1 Atm (dry air with 450 ppm CO_2) Eq(81)/Eq(84)		
t°C (a)	(nr)	(nr)	(nr)	0°C	+15°C	+30°C
2-3	6562.85	-	6562.793	6562.720 434	6562.819 964	6562.909 648
2-4	4861.36	-	4861.326	4861.264 784	4861.339 338	4861.406 516
2-5	4340.46	4340.465	4340.467	4340.409 354	4340.476 348	4340.536 714
2-6	4101.74	4101.731	4101.738	4101.683 502	4101.747 054	4101.804 319
2-7	3970.07	3970.073	3970.075	3970.022 424	3970.084 087	3970.139 649
2-8	3889.05	3889.064	3889.051	3889.000 161	3889.060 663	3889.115 180
2-9	3835.39	3835.397	-	3835.336 036	3835.395 772	3835.449 598
2-10	-	3797.910	-	3797.850 053	3797.909 253	3797.962 597
2-11	-	3770.634	-	3770.582 913	3770.641 725	3770.694 718
2-12	-	3750.152	-	3750.104 730	3750.163 250	3750.215 981
2-13	-	3734.371	-	3734.321 147	3734.379 443	3734.431 972
2-14	-	3721.948	-	3721.891 575	3721.949 694	3722.002 063
2-15	-	3711.980	-	3711.924 169	3711.982 146	3712.034 387
2-16	-	3703.861	-	3703.806 215	3703.864 077	3703.919 214
2-17	-	3697.159	-	3697.105 112	3697.162 879	3697.214 930
2-18	-	3691.553	-	3691.508 169	3691.565 856	3691.617 835
2-19	-	3686.833	-	3686.784 697	3686.842 317	3686.894 236
2-20	-	3682.825	-	3682.761 429	3682.818 992	3682.870 859
2-21	-	3679.372	-	3679.306 135	3679.363 648	3679.415 472
2-22	-	3676.378	-	3676.316 462	3676.373 933	3676.425 719

(a): temperature of air between prism (or grating) and detector, or inside of monochromator.
(nr): not reported

CONCLUDING REMARKS

With help of Archimedes and Ampere, was obtained forgotten formula of Bohr, suggested in letter to Fowler at 15 April 1914, and deduced from mass defect. It appears to be, that this formula is consistent with experimental wavelengths of hydrogen atom spectral lines.

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